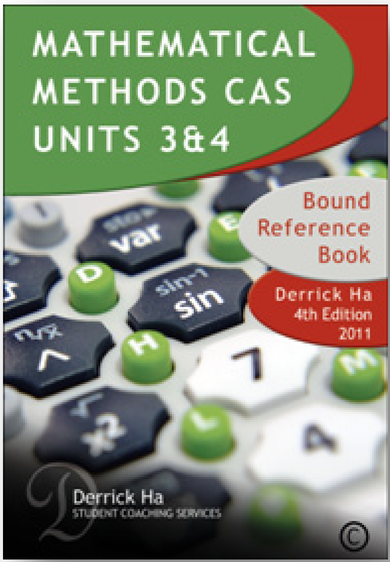
Mathematical Methods Units 3&4

4th Edition



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Preface

In writing this book, it is my hope that you, the student, will discover a new perspective on mathematics. My book aims to complement your teacher's guidance and your regular school textbook by providing a different viewpoint on every topic – the unique viewpoint of a former student who has had success in the VCE and knows the intricacies of what is required.

I aim to provide a more practical explanation of each concept to address the questions that you are likely to have when first learning the material. In this book, you will also find advice that is specific for the VCE, and which will assist you in targeting your learning towards SACs and the end-of-year exams. Other parts of this book will help to improve your understanding by delving into the complexities of each topic so that you can answer the more difficult analysis questions that will inevitably appear in your assessment tasks.

To use this book most effectively, I suggest that you firstly read through it in the same way as you would for a novel – that is, from beginning to end with as few breaks as possible. I have written in a conversational style to create a more interesting read, as well as to provide you with descriptions that are easier to relate to and understand, especially when compared with a regular school text book.

One of my other motivations for writing this book stems from my time as a school student. During my VCE, I found that there were no resources available that showed how to set out a solution for a VCE exam. The textbooks were not handwritten, and neither were the official VCE exam assessment reports. As such, I hope to fill this gap, and have included many hand-written examples to demonstrate the layout of solutions for typical exam questions. These solutions show you the same format that I used during my VCE to much success, and you can use these hand-written answers as a guide for setting out your own working.

The solutions also feature many 'text boxes' and annotations, which illustrate the thinking that I use to write an answer. I am sure that you will have encountered many situations where you understand the solution provided, but do not know why or how to come up with such a solution. The aim of my annotations is to tell you the thought process behind a solution and what I am thinking when I write. This will help to improve your ability to think on your feet and guide your thoughts when tackling more difficult questions.

When you read through this book, you will also encounter many example questions. For the lengthier examples, you should attempt the question before looking at the annotated solution. This will allow you to critique your working, which is important because many students unwittingly lose marks and waste time as a result of incorrect or disorganised 'working out'.

As you probably know, revision throughout the year is very important, and I am sure that many of you will not revise as often as you should. Therefore, I have placed a set of revision questions after every few chapters. These questions are written in exactly the same style as you will encounter in the end-of-year exams, and so will help you to adjust to the unique format of VCE exam questions. They will also assist you with beginning your exam preparation earlier in the year. For these exam questions, I provide an indication of the difficulty level so that you can measure your progress, especially in terms of where you need to be to succeed in your final exams.

I have also included four Trial Exams for later in the year. With the sheer number of exams available to students from both VCAA and other companies, I feel that it would be pointless for me to write another ‘standard exam’. There are already many easy and ‘standard’ exams available, and you should definitely attempt these first when you begin your exam preparation. However, you will eventually reach a stage when you want to extend yourself to prepare for those more difficult VCE exam questions. It is then, that you will be ready to tackle the trial exams that I have written.

My trial exams relate directly to the current 2011 VCE syllabus, but I have deliberately written questions that are extremely difficult in nature. My exams contain all the tricks and traps that I can think of, and are based on my own experiences during the VCE, as well as those from my teaching and lecturing. So, don’t be disheartened by unsuccessful attempts at my exams – rather, learn from them, so that you gain the valuable insight and know the potential pitfalls when attempting similar questions in the future.

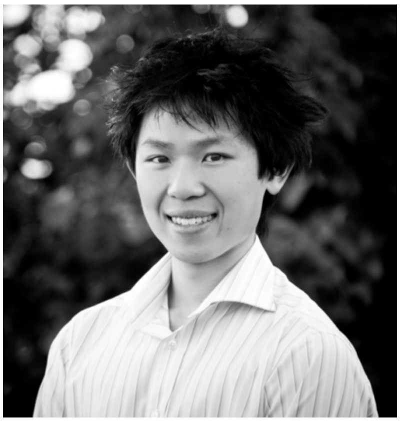
Finally, at the end of this book, you will find space for your own notes. I have written this book with you, the student, in mind. You will need a bound reference book for your SACs and exams, and it would be a waste of time for you to write your own notes. I want you to use this book as your reference book, but understand that you may also want to add your own notes. This space will allow you to do so.

For now, begin reading, writing and working through this student guide. If you can absorb all of the concepts, tips and suggestions, I am sure that you will soon be on a path towards mathematical success.

I wish you all the best for your studies.

Derrick Ha
April 2011

About the Author



Over the past three years, Derrick has established himself as an author, tutor and lecturer in senior VCE mathematics. Since founding Derrick Ha Student Coaching Services in 2008, Derrick has assisted more than a thousand students with their VCE and helped them to achieve their goals.

Derrick's unique teaching style is drawn from his personal experiences of the VCE and his active extracurricular involvement. In 2007, Derrick attained the top ENTER (ATAR) score of 99.95, with the following results:

	Subject	Raw Study Score	Year
Top 4 Subjects	English Language	50	2007 (Year 12)
	Specialist Mathematics	50 (scaled to 54.2)	2007
	Accounting	50	2006 (Year 11)
	Mathematical Methods	50	2005 (Year 10)
5th Subject	Chemistry	50 (5.0 increment)	2006
6th Subject	University Mathematics	High Distinction (5.5 increment)	2007
		Aggregate = 214.7	
Extra 7th Subject	English	48	2007

Derrick’s achievements also extend outside of the VCE. He has had many successes in mathematics competitions and, in 2007, achieved a perfect score in the Westpac Australian Mathematics Competition. He was awarded a gold medal and the B H Neumann Certificate for being the only Senior student in Australasia to achieve this perfect score. He was also an invited attendee of four training selection schools for the Australian Mathematical Olympiad Team. In both 2004 and 2005, Derrick was awarded a Diploma by the Russian Academy of Sciences for his accomplishments in the International Tournament of Towns Mathematics Competition.

Derrick also has extensive experience in public speaking and mathematics coaching. He is the sole lecturer for state-wide end-of-year revision lectures for both Mathematical Methods CAS and Specialist Mathematics. These lectures have been held annually since 2008. Derrick is also a guest speaker and tutor, and has previously volunteered to teach English and mathematics to Sudanese immigrants.

His accomplishments as an orator include being a speaker in the team that reached the DAV Debating State Finals in four separate years. He also experienced success in mock-law courts, as a speaker in the legal team that won the State Mooting Titles in the 2007 Bond University Mooting Competition.

In his VCE year, Derrick was the School Vice-Captain of Haileybury College and the Firsts Team Badminton Captain. He is a recipient of the VCE Premier’s Award Top All-Round Achiever, the Australian Student Prize and the Monash University Prize for Academic Excellence in Year 11.

Derrick is currently studying medicine at the University of Melbourne, as a recipient of the National Medicine Full Scholarship.

Sample – Hand-Written Solution

This is an example of the hand-written solutions provided in the book. These are designed to illustrate Derrick's thought process, as well as showing how you can set out your own working in the VCAA exam.

Question:

Solve for x, where $\log_2(2x+1)+1 \leq 5$.

Hand-Written Solution:

(Continued on next page)

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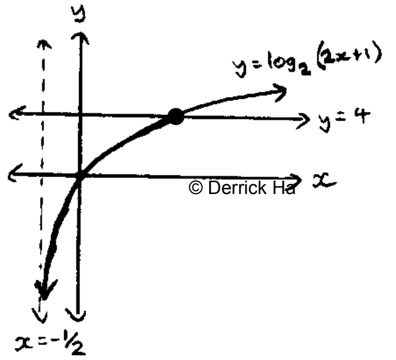

1 $\log_2(2x+1) + 1 \leq 5$

2 $\log_2(2x+1) \leq 4$

Consider the graphs of $y=4$ and $y = \log_2(2x+1)$:

4 Find intersection:

$$4 = \log_2(2x+1)$$

$$2^4 = 2x+1$$

$$2x = 15$$

5 $$x = 15/2$$

6 $\therefore$ From graph: $\underbrace{-\frac{1}{2} <}_{\text{ie. due to asymptote}} x \leq \frac{15}{2}$

1 – Firstly, try to rearrange the inequality as if you were solving for x. Here, we do this by subtracting "1" from both sides.

Stop when you reach the stage where you need to convert from logarithms to exponentials (or vice versa). We reach this stage at (2)

2 – Here, in order to solve for x, we need to convert from logarithms to exponentials. This is problematic because after we do this conversion, it is difficult to determine which way the inequality sign should point. Furthermore, if we converted to exponentials, we would be disregarding the effects of any asymptotes (we will see later that asymptotes affect the answer).

To avoid these problems, and to simplify the question, we draw a graph.

3 – Here, we sketch the graphs of y = LHS and y = RHS. These graphs are useful because they allow the inequality to be seen visually. That is, from (2), we know that the logarithm needs to be less than or equal to 4. On the graph, this is indicated by the thickened region of the log graph. Note the closed circle at the intersection of the two graphs – it is closed because we have a less than **or equals to** sign in the inequality.

By looking at the thickened part of the log graph, we can see that it corresponds to a set of x-values that are to the left of the intersection point. In other words, our final answer for the entire question is going to be $x < something$.

We need to find the value of this 'something' – that is, we need to find the x-value at the intersection of the two graphs.

4 – Find the intersection point – note how we do not need to worry about any inequalities when finding this point (this is why the graphing method is easier).

5 – This is the x-value at the intersection point. From the graph, we see that the thickened part is to the left of this point. Therefore: $x < 15/2$.

6 – One significant benefit of drawing the graph is that we can see the asymptote at x = -½. Therefore, we also know that $x >$ -½. That is, we cannot have x less than -½ because this would involve taking the logarithm of a negative number.

Sample – Exam Question

This is an example of a question from one of the trial exams in the book. Although not shown in this sample, the book also contains hand-written solutions for all of the exams in order to demonstrate how to set out working in a VCE exam.

Question 1

The 'Great Melbourne Hail Storm' of 2010 is causing a great deal of damage in Melbourne. Two avid mathematicians, Katie and Maggie, are curious about the size of the hail stones during this storm.

In performing their calculations, both girls began with the assumption that all the hail stones were perfectly spherical in shape. They also assumed that each hail stone was solid and made completely of ice, which can melt to form water.

Katie lives in the suburb of Richmond. She determines that the radius of the hail stones in this suburb is normally distributed, with an average radius of 30mm and a standard deviation of 5mm.

a) Katie randomly picks up a hail stone from the ground. What is the probability that it has a diameter greater than 60mm?

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1 mark

Katie notices another hail stone which seems to be particularly large. She picks it up and finds that it initially has a radius of 50mm.

b) Show that the initial volume, V millilitres, of this hail stone is $\frac{500\pi}{3}$.

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2 marks

When outdoors, Katie approximates that the rate at which a hail stone will melt is proportional to its surface area. She writes this down as the following mathematical equation:

$$\frac{dV}{dt} = k \cdot A$$

where:

- $k \in R$
- A is the surface area of the sphere in square centimetres
- t is the time elapsed in minutes since a hail stone was picked up

c) State any relevant restrictions on the value of k. Justify your answer.

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2 marks

During the first minute, the hail stone which initially had a radius of 50mm, melts significantly and loses $\frac{496\pi}{3}$ millilitres of volume. However, during the entire time, it maintains its spherical shape.

d)

i) Show that $\frac{dV}{dt} = k \cdot 6^{\frac{2}{3}} \pi^{\frac{1}{3}} \cdot V^{\frac{2}{3}}$.

2 marks

ii) By first considering an appropriate integral, find the value of *k*.

5 marks

iii) Hence find the time taken for the volume of the hail stone to decrease to 1 ml. Give your answer correct to two decimal places.

2 marks

Maggie lives in Blackburn, which is another suburb in Melbourne. Unfortunately, here, the hail stones are larger. As another astute mathematician, Maggie would like to determine the mean and standard deviation of the radius of the hail stones in Blackburn. Like Katie, she assumes that the hail stones are normally distributed in size.

Maggie finds that the average radius of the hail stones is 40mm. She also determines that 90% of the hail stones have a radius of less than 50mm.

e) Show that, correct to 1 decimal place, the standard deviation for the hail stone radius in Blackburn is 7.8. You should use this rounded value for the remaining parts of this question.

2 marks

Maggie is watching her neighbour's car, which is unfortunately being subjected to a number of large hail stones. Large hail stones have the potential to break car windows. In order to do so, the window must first be weakened by a hail stone that is larger than 50mm in radius. **Only once it is weakened** can the window be broken by a second hail stone. This second hail stone needs to be at least 40mm in radius to break the weakened window.

Maggie notices that the neighbour's car window is still intact and is not weakened. However, over the next 30 seconds, three hail stones hit the car window.

f) What is the probability that the car window is not broken after the 30 seconds? Give your answer correct to two decimal places.

5 marks

Total 21 marks