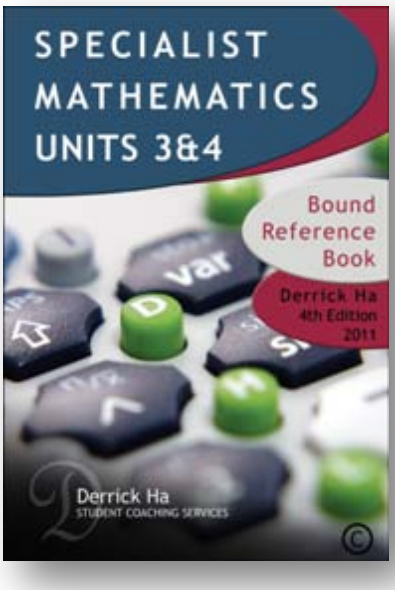
Specialist Mathematics Units 3&4
4th Edition



261 pages
ISBN 978-0-9871258-1-1

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Preface

In writing this book, it is my hope that you, the student, will discover a new perspective on mathematics. My book aims to complement your teacher's guidance and your regular school textbook by providing a different viewpoint on every topic – the unique viewpoint of a former student who has had success in the VCE and knows the intricacies of what is required.

I aim to provide a more practical explanation of each concept to address the questions that you are likely to have when first learning the material. In this book, you will also find advice that is specific for the VCE, and which will assist you in targeting your learning towards SACs and the end-of-year exams. Other parts of this book will help to improve your understanding by delving into the complexities of each topic so that you can answer the more difficult analysis questions that will inevitably appear in your assessment tasks.

To use this book most effectively, I suggest that you firstly read through it in the same way as you would for a novel – that is, from beginning to end with as few breaks as possible. I have written in a conversational style to create a more interesting read, as well as to provide you with descriptions that are easier to relate to and understand, especially when compared with a regular school text book.

One of my other motivations for writing this book stems from my time as a school student. During my VCE, I found that there were no resources available that showed how to set out a solution for a VCE exam. The textbooks were not handwritten, and neither were the official VCE exam assessment reports. As such, I hope to fill this gap, and have included many hand-written examples to demonstrate the layout of solutions for typical exam questions. These solutions show you the same format that I used during my VCE to much success, and you can use these hand-written answers as a guide for setting out your own working.

The solutions also feature many 'text boxes' and annotations, which illustrate the thinking that I use to write an answer. I am sure that you will have encountered many situations where you understand the solution provided, but do not know why or how to come up with such a solution. The aim of my annotations is to tell you the thought process behind a solution and what I am thinking when I write. This will help to improve your ability to think on your feet and guide your thoughts when tackling more difficult questions.

When you read through this book, you will also encounter many example questions. For the lengthier examples, you should attempt the question before looking at the annotated solution. This will allow you to critique your working, which is important because many students unwittingly lose marks and waste time as a result of incorrect or disorganised 'working out'.

As you probably know, revision throughout the year is very important, and I am sure that many of you will not revise as often as you should. Therefore, I have placed a set of revision questions after every few chapters. These questions are written in exactly the same style as you will encounter in the end-of-year exams, and so will help you to adjust to the unique

format of VCE exam questions. They will also assist you with beginning your exam preparation earlier in the year. For these exam questions, I provide an indication of the difficulty level so that you can measure your progress, especially in terms of where you need to be to succeed in your final exams.

I have also included four Trial Exams for later in the year. With the sheer number of exams available to students from both VCAA and other companies, I feel that it would be pointless for me to write another 'standard exam'. There are already many easy and 'standard' exams available, and you should definitely attempt these first when you begin your exam preparation. However, you will eventually reach a stage when you want to extend yourself to prepare for those more difficult VCE exam questions. It is then, that you will be ready to tackle the trial exams that I have written.

My trial exams relate directly to the current 2011 VCE syllabus, but I have deliberately written questions that are extremely difficult in nature. My exams contain all the tricks and traps that I can think of, and are based on my own experiences during the VCE, as well as those from my teaching and lecturing. So, don't be disheartened by unsuccessful attempts at my exams – rather, learn from them, so that you gain the valuable insight and know the potential pitfalls when attempting similar questions in the future.

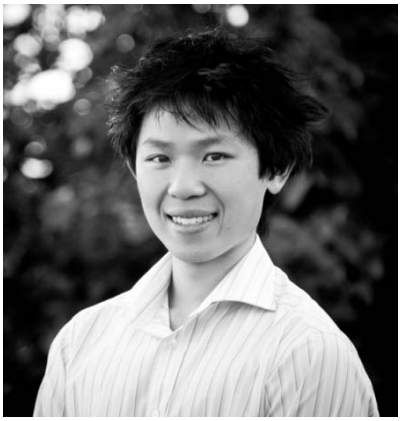
Finally, at the end of this book, you will find space for your own notes. I have written this book with you, the student, in mind. You will need a bound reference book for your SACs and exams, and it would be a waste of time for you to write your own notes. I want you to use this book as your reference book, but understand that you may also want to add your own notes. This space will allow you to do so.

For now, begin reading, writing and working through this student guide. If you can absorb all of the concepts, tips and suggestions, I am sure that you will soon be on a path towards mathematical success.

I wish you all the best for your studies.

Derrick Ha
April 2011

About the Author



Over the past three years, Derrick has established himself as an author, tutor and lecturer in senior VCE mathematics. Since founding Derrick Ha Student Coaching Services in 2008, Derrick has assisted more than a thousand students with their VCE and helped them to achieve their goals.

Derrick's unique teaching style is drawn from his personal experiences of the VCE and his active extracurricular involvement. In 2007, Derrick attained the top ENTER (ATAR) score of 99.95, with the following results:

	Subject	Raw Study Score	Year
Top 4 Subjects	English Language	50	2007 (Year 12)
	Specialist Mathematics	50 (scaled to 54.2)	2007
	Accounting	50	2006 (Year 11)
	Mathematical Methods	50	2005 (Year 10)
5th Subject	Chemistry	50 (5.0 increment)	2006
6th Subject	University Mathematics	High Distinction (5.5 increment)	2007
		Aggregate = 214.7	
Extra 7th Subject	English	48	2007

Derrick's achievements also extend outside of the VCE. He has had many successes in mathematics competitions and, in 2007, achieved a perfect score in the Westpac Australian Mathematics Competition. He was awarded a gold medal and the B H Neumann Certificate for being the only Senior student in Australasia to achieve this perfect score. He was also an invited attendee of four training selection schools for the Australian Mathematical Olympiad Team. In both 2004 and 2005, Derrick was awarded a Diploma by the Russian Academy of Sciences for his accomplishments in the International Tournament of Towns Mathematics Competition.

Derrick also has extensive experience in public speaking and mathematics coaching. He is the sole lecturer for state-wide end-of-year revision lectures for both Mathematical Methods CAS and Specialist Mathematics. These lectures have been held annually since 2008. Derrick is also a guest speaker and tutor, and has previously volunteered to teach English and mathematics to Sudanese immigrants.

His accomplishments as an orator include being a speaker in the team that reached the DAV Debating State Finals in four separate years. He also experienced success in mock-law courts, as a speaker in the legal team that won the State Mooting Titles in the 2007 Bond University Mooting Competition.

In his VCE year, Derrick was the School Vice-Captain of Haileybury College and the Firsts Team Badminton Captain. He is a recipient of the VCE Premier's Award Top All-Round Achiever, the Australian Student Prize and the Monash University Prize for Academic Excellence in Year 11.

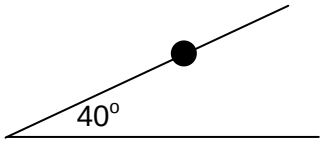
Derrick is currently studying medicine at the University of Melbourne, as a recipient of the National Medicine Full Scholarship.

Sample – Hand-Written Solution

This is an example of the hand-written solutions provided in the book. These are designed to illustrate Derrick's thought process, as well as showing how you can set out your own working in the VCAA exam.

Question:

A 6 kg mass is placed on a rough slope as shown (μ = 0.4). Find the acceleration of the mass (to 2 decimal places), and determine how far it has travelled from rest after 3 seconds.



Hand-Written Solution:

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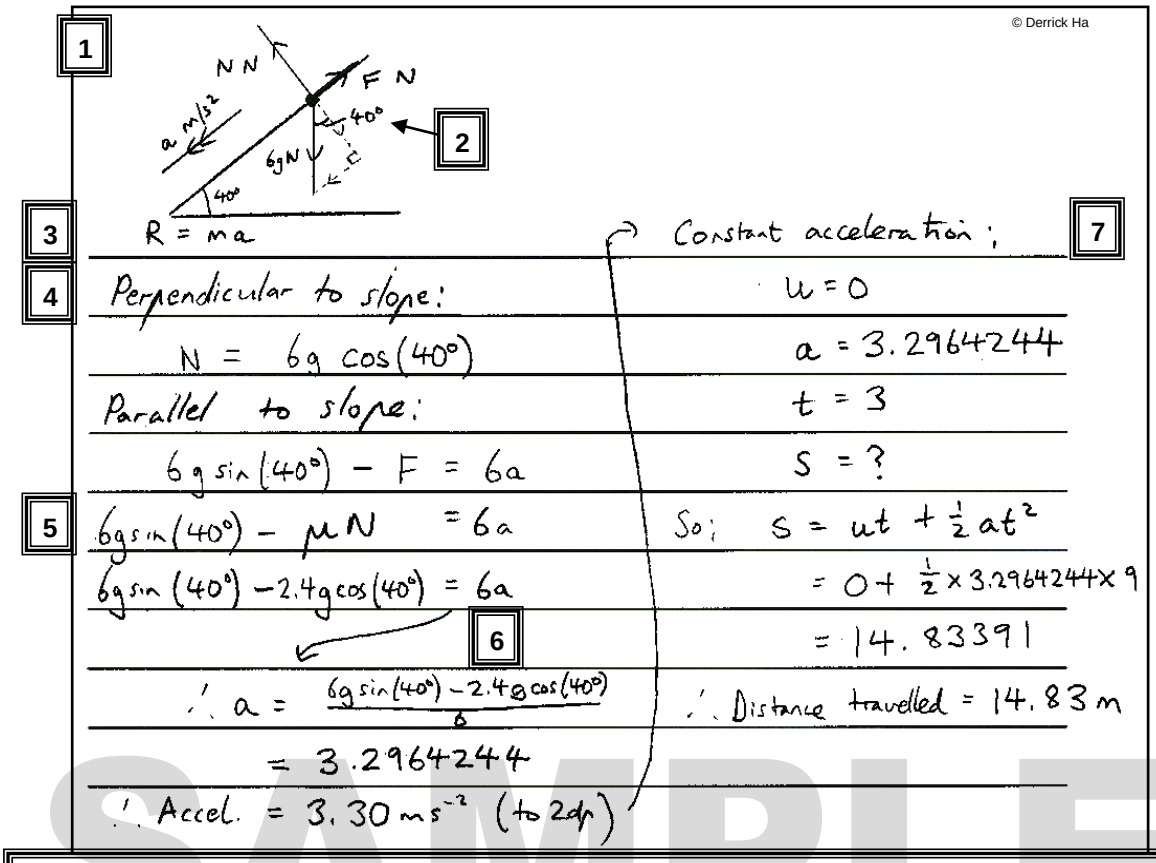
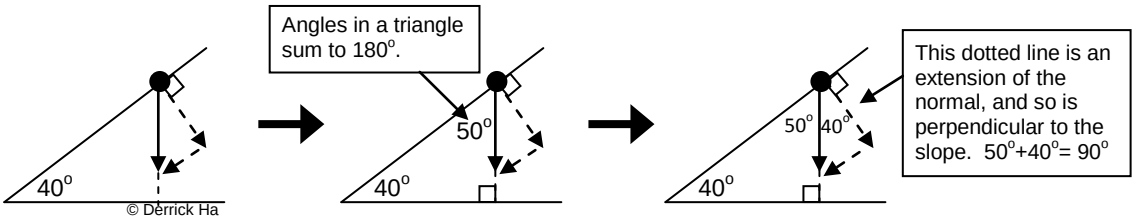


1 – Always begin your working with a force diagram. Show the units. Make sure that you define the direction of acceleration – this is usually done with a double arrow (as shown).

2 – The working which proves that this is 40^o is never shown (it is unnecessary). However, you must understand how to work it out. Quite simply, use the properties of a right angled triangle:



3 – I always state this equation immediately after my force diagram. It is the basis of the rest of my working and, by stating it once at the start, I don't need to keep on stating it every time I use it.

4 – For each object, it is usually (but not always) useful to resolve parallel and perpendicular to the direction of acceleration.

5 – Here, I can assume that F = μN because the question implies that the particle is moving (you should remember that F only equals μN when a particle is moving or on the point of moving). With direct reference to the question, it is stated that "determine how far...seconds". This implies that some distance has been travelled, and therefore we can assume that the particle is moving.

In general, if you assume that F = μN when it actually isn't, then you will obtain a negative value for the acceleration. This 'negative acceleration' would be incorrect and illogical. Therefore, if you obtain a 'negative acceleration', then you should always double-check your working.

6 – Always keep your 'equal signs' directly above each other. When I need to move it across, I use an arrow. This may seem pedantic, but it helps to make your working much clearer for the examiner.

7 – Refer to previous section on 'Constant Acceleration' for explanation of this working.

Sample – Exam Question

This is a question from one of the trial exams in the book. Although not shown in this sample, the book also contains hand-written solutions for all of the exams in order to demonstrate how to set out working in a VCE exam.

Question 1
Sahil is a very talented chemistry student. One day, he is conducting an experiment to investigate the concentration of coloured dye when it is added to water.

For the first part of the experiment, Sahil takes a 100L tank (Tank A), which is initially filled with 100L of pure water. To begin the experiment, he simultaneous does three things to the tank:

- A colour dye solution with a concentration of 5 units per litre is added to Tank A at a rate of 3L per minute.
- Tank A is continuously well mixed.
- A hole is created in the bottom of Tank A, allowing the tank mixture to be removed at a rate of 3L per minute.

The experiment is continued for a period of 10 minutes. At the 10 minute mark, the experiment is ceased so that no more dye solution is added and the hole in the bottom of Tank A is sealed.

Let D be the total amount of dye in the tank (in units) at any time t minutes.

a) Show that the rate of change of dye in the tank can be given by $\frac{dD}{dt} = 15 - \frac{3D}{100}$.

1 mark

b) Using calculus, hence calculate the amount of dye in the tank at any time t.

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4 marks

c) If the experiment were continued indefinitely, instead of only 10 minutes, what value would D approach? Justify your answer.

2 marks

In preparation for the second part of the experiment, Sahil cleans out Tank A and resets everything to its original state, so that Tank A again contains 100L of fresh water.

For this second part of the experiment, Sahil spends another 10 minutes repeating **exactly the same** experiment as for the first part with **only one vital change**. He now collects the mixture flowing out of Tank A and allows it to flow into another large container (Tank B). Tank B initially contains 10L of water with 10 units of dissolved dye.

Tank B is also continuously well mixed, and has a hole in the bottom. From this hole, the mixture of Tank B is removed at a rate of 2 litres per minute. After 10 minutes, this second experiment is ceased completely so that there is no more inflow or outflow for any of the tanks.

Let X be the total amount of dye in Tank B (in units) at any time t minutes. You should consider $t = 0$ to be the start of the second part of the experiment.

d) Find a differential equation for the rate of change of dye in tank B. Give your differential equation in terms of only t and X.

4 marks

e) Using Euler's Method with a step size of 0.5, approximate the amount of dye in Tank B one minute after the start of the second part of the experiment. Give your answer correct to three decimal places.

2 marks

Despite his talent, Sahil is often very clumsy during chemistry experiments. Whilst cleaning up after the experiment, he accidentally turns the tap the wrong way. As a result, water gushes out of the tap and drenches the wall of the chemistry classroom.

A fellow student, Satwik, examines one of the water droplets on the wall and notices that it takes 5 seconds to travel a distance of one metre in a straight line down the wall. At the end of this one metre distance, the water droplet has a speed of 30 cm per second.

f) Assuming that the water droplet experiences constant acceleration, calculate the exact acceleration of the water droplet in ms^{-2}.

3 marks

Total 16 marks